Period 1, Jan 13, 2024

8. Use the limit to find the horizontal asymptotes of $g(t) = \frac{3t^2+2}{\sqrt{t^2+4}}$

$$g'(x) = \frac{6T(T^2+4)^{\frac{1}{2}} - (3T^2+2)\left(\frac{1}{2}(T^2+4)^{-\frac{1}{2}}(2T)\right)}{T^2+4}$$

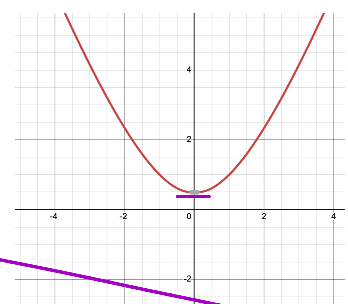
$$0 = \frac{\left[6T\sqrt{T^2+4} - \frac{(3T^2+2)(\cancel{2}T)}{\cancel{2}\sqrt{T^2+4}}\right]\left(\sqrt{T^2+4}\right)}{T^2+4 \quad \left(\sqrt{T^2+4}\right)} = \frac{6T(T^2+4) - T(3T^2+2)}{(T^2+4)\left(\sqrt{T^2+4}\right)}$$

$$6T(T^2+4) - T(3T^2+2) = 0$$

$$6T^3 + 24T - 3T^3 - 2T = 0$$

$$3T^3 + 22T = 0 \Rightarrow T(3T^2+22) = 0$$

$$T = 0$$

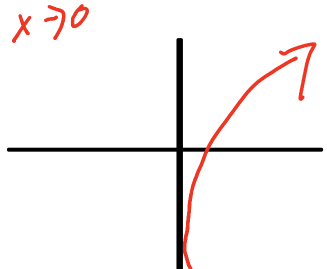


c) $\lim_{x \to 0^+} x(\ln x)$

(Tip: think about the graph of $\ln x$ at $x = 0$)

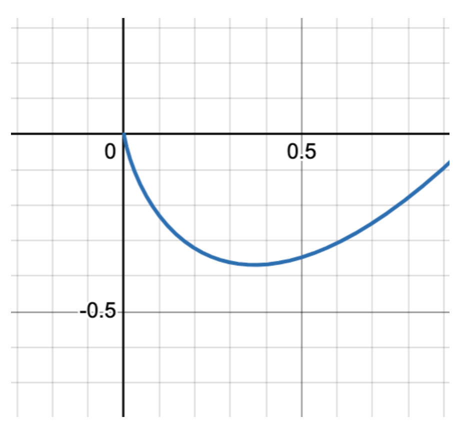
$Y = \ln x$

$\lim_{x \to 0} \ln x = -\infty$



$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} = \frac{-\infty}{\infty} = 0$$

$$\lim_{x \to 0^+} \frac{\ln x}{x^{-1}} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-1x^{-2}} = \frac{\frac{1}{x}}{\frac{-1}{x^2}} = -x$$



4. Find the differential for $y = \cos^2 x$ $\Rightarrow y = u^2$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$\frac{dy}{du} = 2u$

$\frac{du}{dx} \cdot \frac{dy}{du} = \frac{dy}{dx}$

$-\sin x \cdot 2u$

$\frac{dy}{dx} = -\sin x \cdot 2\cos x$

$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = -2\sin x \cos x \cdot dx$

$dy = -2\sin x \cos x\, dx$

3. Fill out the information in the table. Using this information to sketch the graph of the function.
$f(x) = 2x^3 + 4x^2 + 2x$
Show your work here:

x-intercept (x,y)	
y-intercept (x,y)	
Critical points (x-only)	
Interval of increase	$(-\infty, -1) \cup (-\frac{1}{3}, \infty)$
Interval of decrease	$(-1, -\frac{1}{3})$
Relative Minimum	
Relative Maximum	
Point(s) of inflection (x-only)	
Interval of Concave up	
Interval of Concave down	

increase $\frac{dy}{dx} = +$

decrease $\frac{dy}{dx} = -$

$F'(x) = 6x^2 + 8x + 2$

$F'(x) = 2(3x^2 + 4x + 1) = 2(3x^2 + 3x + 1x + 1)$

$3 \cdot 1 = 3$ $\quad 3 + 1 = 4$

$3x(x+1) \quad 1(x+1)$

$= 2(x+1)(3x+1) = F'(x)$

$F'(x) = 0 \Rightarrow x = -1, -\frac{1}{3}$

++++ $\quad$ ---- $\quad$ ++++++

$-2 \quad -1 \quad -\frac{1}{3} \quad 0 \quad 1 \quad 2$

$F'(-2) = 2 \cdot -1 \cdot -5 = +$ $\quad F'(0) = 2 \cdot 1 \cdot 1 = +$

$F'(-\frac{1}{2}) = 2 \cdot \frac{1}{2} \cdot -\frac{1}{2} = -$

4. Given $m(t) = -2\sin(2t)$; $[-\pi, \pi]$, find the open intervals where the function is concave up and concave down. Justify your answer.

$\sin -\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos -\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\sin -\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$m(x) = -2\sin 2x \Rightarrow y = -2\sin u$

$u = 2x$

$\frac{du}{dx} = 2$

$\frac{dy}{du} = -2\cos u$

$\sin -\pi = 0$ $\quad$ $\sin 0 = 0$

$\cos -\frac{\pi}{2} = 0$ $\quad$ $\cos \frac{\pi}{2} = 0$

$\sin \pi = 0$

$m'(x) = -4\cos 2x$

$m''(x) = 4 \cdot 2\sin 2x$

$= 8\sin 2x = 8 \cdot 2\sin x\cos x = 16\sin x\cos x$

+ + + $\quad$ - - - - $\quad$ up

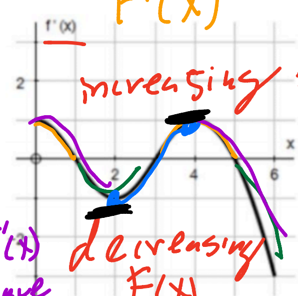
+ · - · + $\quad$ + + +

$-\pi$ $\quad$ up $\quad$ $-\frac{3\pi}{4}$ $\quad$ $-\pi/2$ $\quad$ down $\quad$ $-\frac{\pi}{4}$ $\quad$ 0 $\quad$ $\frac{\pi}{4}$ $\quad$ $\frac{\pi}{2}$ $\quad$ $\frac{3\pi}{4}$ $\quad$ π

5. Use the graph of $f'(x)$ defined over the interval [0, 6] provided below to answer the following.

a) When is $f(x)$ increasing? When is $f(x)$ decreasing? Justify your responses.

$(0,1)\cup(3,5)$ increasing $\quad$ $(1,3)\cup(5,6)$ decreasing



b) Determine the x-coordinates of all local extrema. Justify your response.

$x = 1, 3, 5$ $\quad$ $x = 1$ max $\quad$ $x = 5$ max

$x = 3$ min

c) When is f concave up? When is f concave down? Justify your responses.

$F''(x) = + =$ slope of $F'(x)$ $\quad$ $(2,4)$ concave up

$F''(x) = - =$ slope of $F'(x)$ $\quad$ $(0,2)\cup(4,6)$ concave Down

increasing $F'(x) = +$

decreasing $F'(x) = -$

d) Find the x-coordinates of all points of inflection. Justify your response.

$F''(x) = 0$ or ϕ

change of concavity

$F''(x) =$ slope of $F'(x)$

$x = 2$ $\quad$ slope from $-$ to $+$ $\quad$ $F''(x)$ from $-$ to $+$

$x = 4$ $\quad$ slope from $+$ to $-$ $\quad$ $F''(x)$ from $+$ to $-$

Example 1: Use Optimization techniques to find two numbers whose sum is 20 and whose product is as large as possible. What is the largest product?

$F'(x) = 0$ or ϕ max or min

Biggest m possible

$x + y = 20$

$y = 20 - x$

$x \cdot y = m$

$x(20 - x) = m$

$20x - x^2 = m$

$20 - 2x = \frac{dm}{dx}$

$20 - 2x = 0$

$20 = 2x$

$10 = x$

$10 + 10 = 20$

$10 \cdot 10 = 100$

$9 + 11 = 20$

$9 \cdot 11 = 99$

Max

$10 + 10$

$10 \cdot 10 = 100$

Min

No min

$\cancel{0 + 20}$

$0 \cdot 20 = 0$

$21 + -1 = 20$

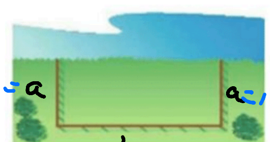
$21 \cdot -1 = -21$

Example 2: You have 40 feet of fence to enclose a rectangular garden along the side of a lake. What is the maximum area that you can enclose?

y'

Area = $a \cdot b$

$y = a \cdot b$



$2a + b = 40$

$b = 40 - 2a$

$y = a(40 - 2a)$

$y = 40a - 2a^2$

$\frac{dy}{da} = 40 - 4a = 0$

$40 = 4a$

$a = 10$ Max

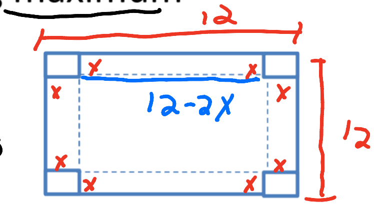
$y = 40a - 2a^2$

$\frac{dy}{da} = 40 - 4a$

$\frac{d^2y}{da^2} = -4$ Concave Down

Max

Example 3: An open-top box is to be made by cutting congruent squares of sides $\boldsymbol{x}$ from the corners of a 12- by 12-cm sheet of tin and bending up the sides. How large should the squares be to make the box hold as much as possible? What is the resulting maximum volume? $X = 2$ $\quad 12-4=8$



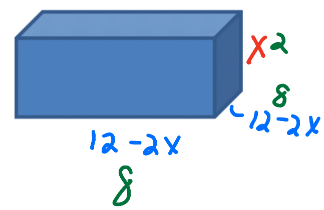
$$V = x(12-2x)(12-2x) = x(12-2x)^2 = 144x - 48x^2 + 4x^3$$

$$\frac{dV}{dx} = 144 - 96x + 12x^2 = 12(12 - 8x + x^2)$$

$$12(x^2 - 8x + 12)$$

$$12(x-6)(x-2) = 0$$

$$x = 6 \text{ or } x = 2$$



$$V_{max} = 8 \cdot 8 \cdot 2 = 128 \text{ cm}^3$$

$$\frac{d^2V}{dx^2} = -96 + 24x$$

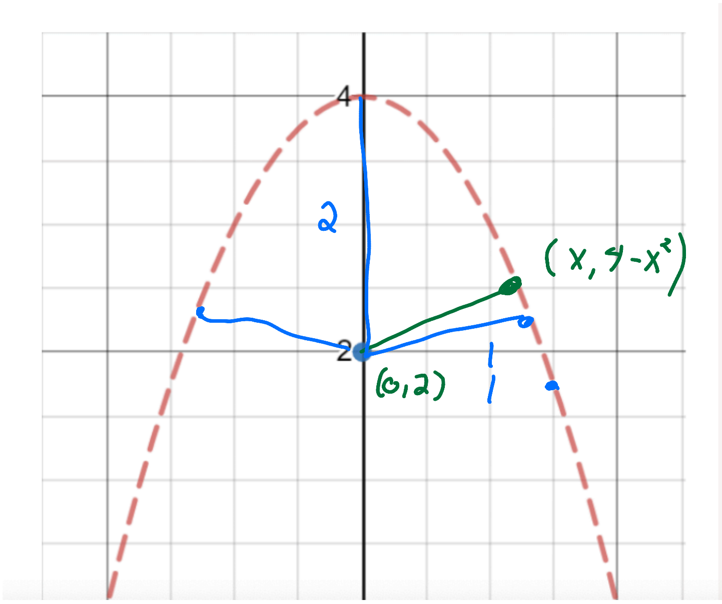
$-96 + 24(6) = -96 + 144 = +$ concave up min

$-96 + 24(2) = -96 + 48 = -$ concave Down Max

$x = 2$

Example 4: Which points of the graph of $y = 4 - x^2$ are closest to the point (0,2)?

$dis = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$d = \sqrt{(x-0)^2 + (4-x^2-2)^2}$

$d^2 = \left(\sqrt{x^2 + (2-x^2)^2}\right)^2$

$d^2 = x^2 + (2-x^2)^2$

$d^2 = x^2 + 4 - 4x^2 + x^4$

$2 \cdot d \cdot \frac{dd}{dx} = \underbrace{2x - 8x + 4x^3}$ $= 0$

$X = \pm\sqrt{\frac{3}{2}}$

d = distance always +

$2x - 8x + 4x^3 = 0$ Then $\frac{dd}{dx} = 0$

$-6x + 4x^3 = 0$

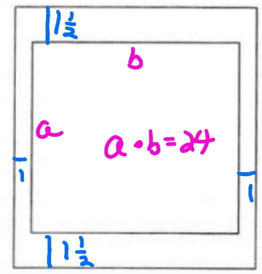
$x(-6 + 4x^2) = 0$

$x = 0$ or $4x^2 = 6$

$x^2 = \frac{3}{2}$

$x = \pm\sqrt{\frac{3}{2}}$

Example 5: A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be 1 ½ inches, and the margins on the left and right are to be 1 inch. What should be the dimensions of the page be so that the least amount of paper is used?



min

$b+1+1$ $\quad a+1\frac{1}{2}+1\frac{1}{2}$

diamensions $4+2$ by $6+3$ or 6 by 9

$(b+2)\cdot(a+3)=$ Area of Paper

$ab=24$ $\quad$ $b=4$

$a=\frac{24}{b}$ $\quad$ $a=\frac{24}{4}=6$

$(b+2)(\frac{24}{b}+3)=A$

$24+3b+48b^{-1}+6$

$A=30+3b+48b^{-1}$

$\frac{dA}{dB}=3-48b^{-2}$

$0=3-\frac{48}{b^2} \Rightarrow b^2\cdot\frac{48}{b^2}=3\cdot b^2$

$\frac{48}{3}=\frac{3b^2}{3}$

$16=b^2$

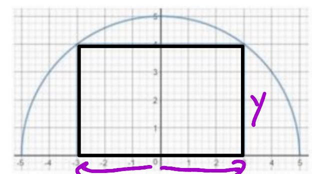
$b=\pm 4$

$b=4$

$\frac{d^2A}{dB^2}=0+96b^{-3}$ always be positive

b=distance

Example 6: A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$. Find the dimensions of the rectangle that would maximize the area of the rectangle.



$$A_{Area} = 2x \cdot y$$

$$A = 2x(25 - x^2)^{\frac{1}{2}}$$

$$\frac{dA}{dx} = 2(25 - x^2)^{\frac{1}{2}} + 2x\left[\frac{1}{\cancel{2}}(25 - x^2)^{-\frac{1}{2}} \cdot \cancel{2}x\right]$$

$$\phi \text{ or } 0 = \frac{2\sqrt{25 - x^2}}{1} \cdot \frac{\sqrt{25-x^2}}{\sqrt{25-x^2}} + \frac{-2x^2}{\sqrt{25 - x^2}} = \frac{2(25 - x^2) - 2x^2}{\sqrt{25 - x^2}} = \frac{50 - 2x^2 - 2x^2}{\sqrt{25 - x^2}} = \frac{50 - 4x^2}{\sqrt{25 - x^2}}$$

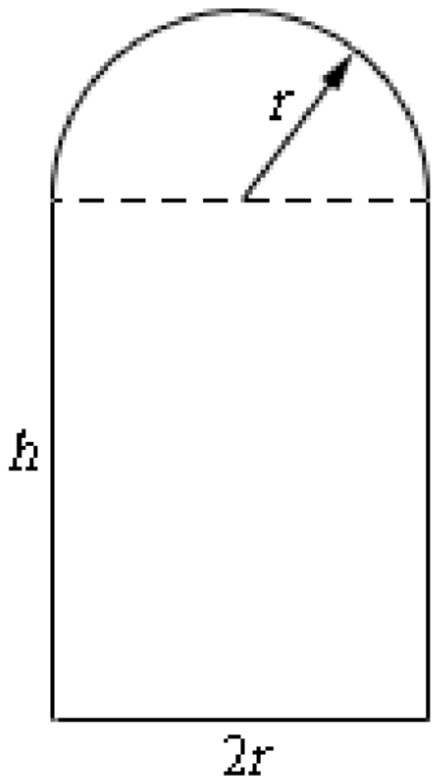
$x = 5$ or -5

$\frac{dA}{dx} = \phi$ Min

Max $50 - 4x^2 = 0$

$\frac{50}{4} = x^2$

$x = \frac{\sqrt{50}}{\sqrt{4}} = \frac{5\sqrt{2}}{2}$



$2\pi r = cir_{circle}$

πr

$h + 2r + h + \pi r = edge$